

Progression of Skills in Number

How we teach our curriculum

At St Mary's, we value each child as an individual learner with his or her own particular needs. Through careful teaching and assessment, teachers identify the next steps in each child's mathematical development and accordingly provide appropriately challenging tasks. This means that children will be taught these strategies at a time that is appropriate to them. Children will need to work through all steps (the 'doing' stage with actual physical resources, the 'seeing' stage by representing maths pictorially and the 'symbolic stage' where they apply their knowledge to written calculations) to fully understand all areas of maths. For those who are new to the term 'CPA' it stands for 'concrete', 'pictorial' and 'abstract'. These terms describe the learning 'building blocks' to deeper mathematical learning. We have used ideas from the Hampshire Maths Hub and adapted them to include everything we do.

Resources/ Materials

At each level we use a range of different resources as we believe that children in all year groups should experience their learning through CPA.

Here are a list of some of the resources we might use:

Toys	Multi- Link	Counters	Small world toys	Bead Strings to 10	Bead Strings to 20
Bead Strings to 50	Bead Strings to 100	Number fans	Base 10	Numicon	Part whole models
Bar models	Dry wipe number lines	Printed number lines	Empty number lines	Number bond cards	Hundred Square

Starting points

Before we start any unit with the children, we will check where they are working.

If this is the first time we are visiting the unit with our class we will carry out a pre-assessment. This gives us an idea of what the children are capable of and what they might be able to move onto next. We will also look at the information given to us by the previous teacher.

After each lesson we record where the children have got to and this will feed into our next lesson. At the end of the unit we will write down what they have achieved and this will then be used the next time we visit our unit. For SEND children we may carry out pre-learning to make sure the children are constantly being reminded of their learning.

On the next pages are the progression of skills for the four operations and will show a variety of different methods we use. The teacher may adapt this to suit their class's needs and use a variety of approaches. These will be linked to the area on this document that they are teaching.


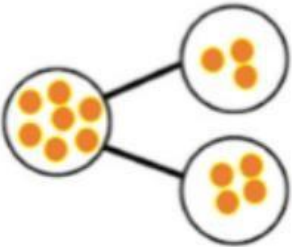
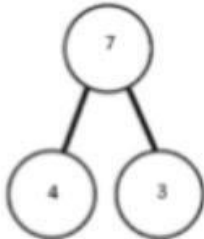
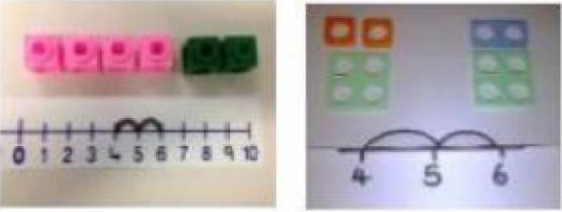
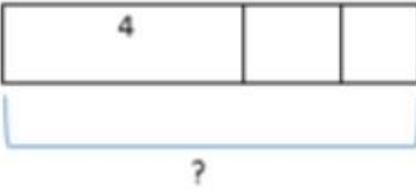

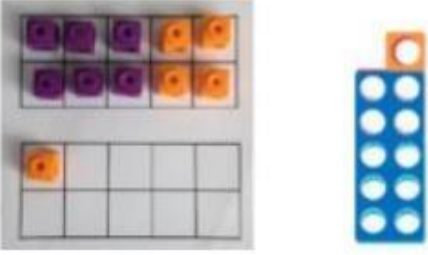
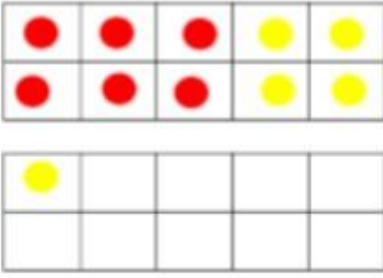
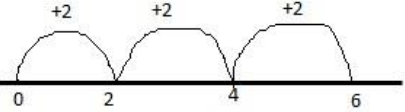
Addition and Subtraction Progression of Skills

Key language: Number bonds, number line, add, more, plus, make, sum, total, altogether, Inverse, double, near, double, half, halve, equals, is the same as (including equals sign), difference, between, How many more to make..?, how many more is...than..?, how much more is..? Subtract, take away, minus, How many fewer is...than..?, How much less is..?, column addition and subtraction, exchanging, efficient written method, order of operations, **Column addition and subtraction**, **Efficient written method**, **Order of operations** **Year 1/2: black words** **Year 3: red words** **Year 4: blue words** **Year 5: green words** **Year 6: Purple words** Expectations

(National Curriculum Statements):

<p>Year R: -Count reliably with numbers from 1-20, place them in order and say which number is one more or one less than a given number -Using quantities and objects add and subtract two single digit numbers and count on or back to find the answer. -Solve problems, including doubling, halving and sharing.</p>		
<p>Year 1</p> <p>-read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs -represent and use number bonds and related subtraction facts within 20 -add and subtract one-digit and two-digit numbers to 20, including zero -solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.</p>	<p>Year 2</p> <p>-solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures, applying their increasing knowledge of mental and written methods -recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 -add and subtract numbers using concrete objects, pictorial representations, and mentally, including: \square a two-digit number and ones \square a two-digit number and tens \square two two-digit numbers \square adding three one-digit numbers -show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot -recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</p>	<p>Year 3</p> <p>- add and subtract numbers mentally, including: a three-digit number and 1s, a three-digit number and 10s, a three-digit number and 100s -add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction -estimate the answer to a calculation and use inverse operations to check answers -solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.</p>
<p>Year 4</p> <p>-add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate - estimate and use inverse operations to check answers to a calculation -solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.</p>	<p>Year 5</p> <p>-add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) -add and subtract numbers mentally with increasingly large numbers -use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy -solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.</p>	<p>Year 6</p> <p>-solve problems involving addition, subtraction, multiplication and division -use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. -solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why -use their knowledge of the order of operations to carry out calculations involving the 4 operations - perform mental calculations, including with mixed operations and large numbers.</p>

Progression of Skills in Addition

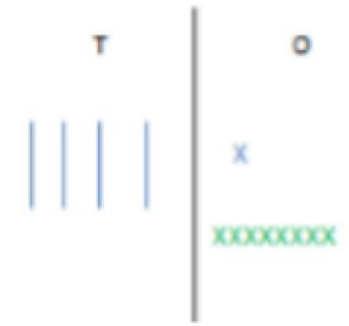
Step	Concrete	Pictorial	Abstract	
1	<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears etc)</p> 		<p>$4 + 3 = 7$ (four is a part, 3 is a part and the whole is seven)</p> 	<p>This might also be shown through addition number sentence with pictures/ objects underneath, Bead strings, number cards, toys etc to be used</p>
2	<p>Counting on using number lines by using cubes or Numicon</p> 	<p>A bar model which encourages the children to count on</p> 	<p>The abstract number line: What is 2 more than 4? What is the sum of 4 and 4? What's the total of 4 and 2? $4 + 2$</p> 	<p>This should build on their idea of what is the 'whole'</p>
3	<p>Regrouping to make 10 by using ten frames and counters/cubes or using Numicon: $6 + 5$</p> 	<p>Children to draw the ten frame and counters/cubes:</p> 	<p>Children to develop an understanding of equality e.g.</p> <p>$6 + \square = 11$ and</p> <p>$6 + 5 = 5 + \square$ $6 + 5 = \square + 4$</p> 	<p>At this point children need a large amount of teaching on what 10 is and what it looks like.(Link to number bonds)</p> <p>We would also teach counting in 2,5,10</p>

4

TO + O using base 10. Continue to develop understanding of partitioning and place value
41 + 8



Children to represent the concrete using a particular symbol e.g. lines for tens and dot/crosses for ones.



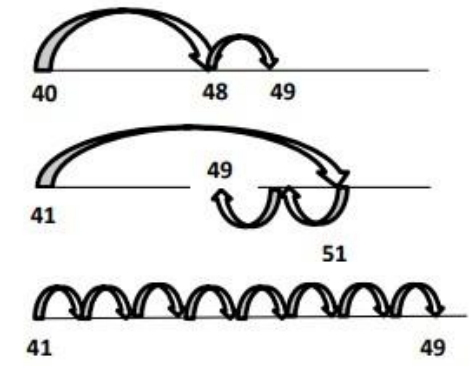
41 + 8 =

Number line:

40 + 8 + 1 =

41 + 10 - 2 =

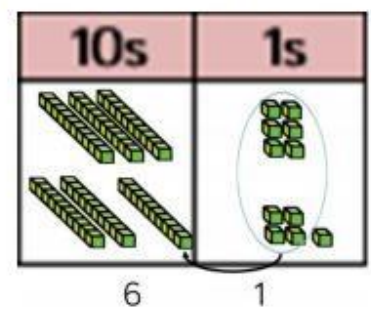
41 + 8 =



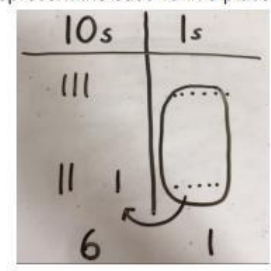
At Broughton we use lots of different techniques on a number line including:
Counting in 1s,
Adding the 10s then the ones, targeting 10. Adding 10 then taking some away if the number is near 10 etc.

5

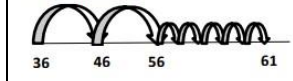
TO + TO using base 10. Continue to develop understanding of partitioning and place value.
36 + 25



Children to represent the base 10 in a place value chart.



a) Number line



b) 'V' method:

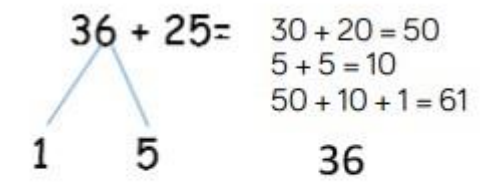
$$\begin{array}{r} 36 + 25 = \\ \text{V} \\ 50 \quad 11 \\ \text{V} \\ 60 \quad 1 = 61 \end{array}$$

c) Column

$$\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ 1 \end{array}$$

Expanded Column	
215	
+176	
11 (5+6)	
80 (10+70)	
300 (200+100)	
<u>391</u>	

Looking for ways to make 10.

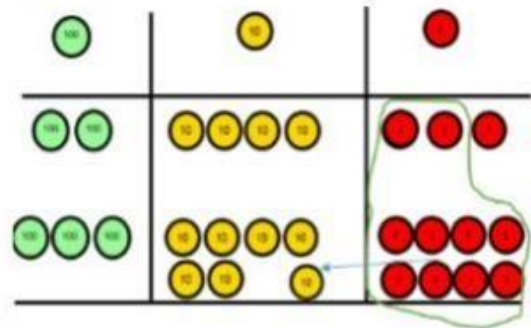


Formal method:

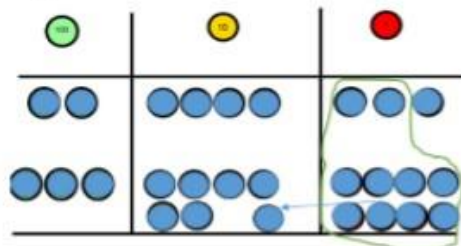
$$\begin{array}{r} +25 \\ 61 \\ \hline 1 \end{array}$$

As this is a key area of learning we practise this in a number of ways including:
Using a tens frame, base 10, number lines, addition facts and then column.

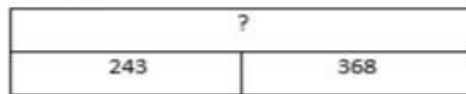
Use of place value counters to add HTO + TO, HTO + HTO etc. once the children have had practice with this, they should be able to apply it to larger numbers and the abstract



Children to represent the counters e.g. like the image below



If the children are completing a word problem, draw a bar model to represent what it's asking them to do



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 1 \quad 1 \end{array}$$

$$\begin{array}{r} 16.40 \\ +7.68 \\ \hline \hline 24.08 \end{array}$$

After this point the children would try with larger numbers and decimals.

As teachers we understand that there is a range of different ways to ask a question to make sure that children understand the key concepts fully. Below is a table of different ways we might ask an addition question.

Conceptual Understanding: Different ways to ask children to solve 21+34

 	<p>Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?</p> <p>21 + 34 = 55. Prove it</p>	$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$ <p>21 + 34 =</p> <p>Calculate the sum of twenty-one and thirty-four.</p>	<p>Missing digit problems:</p> <table border="1"> <thead> <tr> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> </tr> <tr> <td></td> <td>?</td> </tr> <tr> <td>?</td> <td>5</td> </tr> </tbody> </table>	10s	1s				?	?	5
10s	1s										
	?										
?	5										

Progression of skills in Subtraction

Step	Concrete	Pictorial	Abstract
------	----------	-----------	----------

1 **Physically taking away and removing objects from a whole**
 (ten frames, Numicon, cubes and other items such as beanbags could be used).

$4 - 3 = 1$

Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.

$4 - 3 =$

$\square = 4 - 3$

This might also be shown through subtraction number sentence with pictures/objects underneath, bead strings, number cards, toys etc to be used.

2 **Counting back** (using number lines or number tracks) children start with 6 and count back 2.

$6 - 2 = 4$

Children to represent what they see pictorially e.g.

This should build on their idea of what is the 'whole' We also understand that counting back is a tricky skill, so we make sure the children embed this over time.

3 **Finding the difference** (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.

Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.

Find the difference between 8 and 5.

$8 - 5$, the difference is \square

Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.

At this point children need a large amount of teaching on what 10 is and what it looks like. (Link to number bonds)

At this point we would also teach counting in 2,5,10

4

Making 10 (using numicon or ten frames) $14 - 5$



Children could also do this by subtracting a 5 from the 10.



At this point the children would also start using a dry wipe number line to show subtraction. Children to present the ten frame pictorially



Children to show how they can make 10 by partitioning the subtrahend.

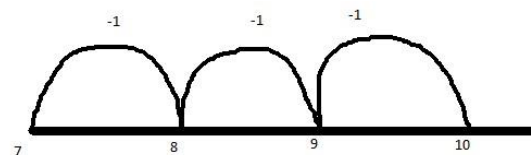
$$14 - 5 = 9$$

$$14 - 4 = 10$$

$$10 - 1 = 9$$

At this time we would also make sure children are confident in finding related facts ($14 - 9 = 5$, $14 = 9 + 5$, $9 + 5 = 14$, $14 - 5 = 9$)

The children will also be showing it on an empty number line.

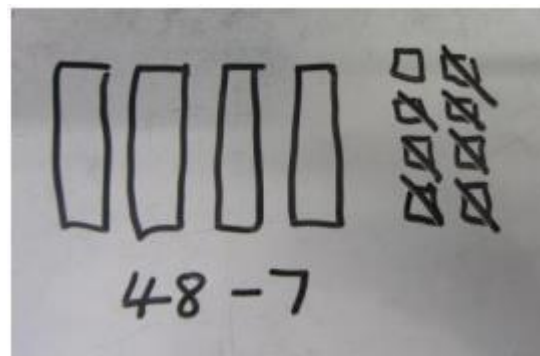


As this is a key area of learning we practise this in a number of ways including: Using a tens frame, base 10, number lines, subtraction facts and then column.

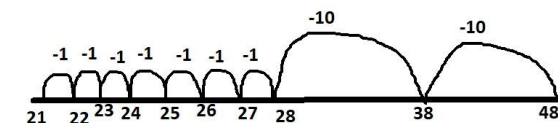
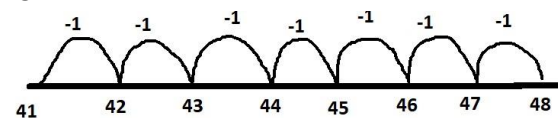
5

Subtracting a single digit from a two-digit number and a two-digit number from a two-digit number (when there is no regrouping) (using base 10)

$$48 - 7$$


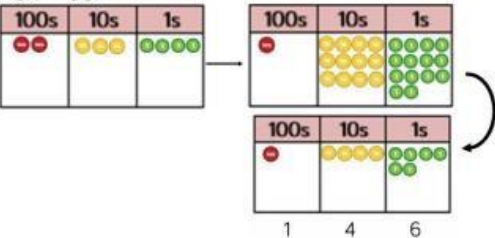
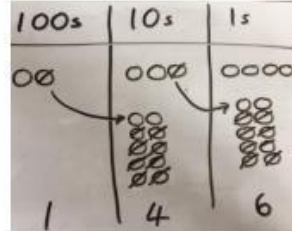


$$48 - 7 =$$



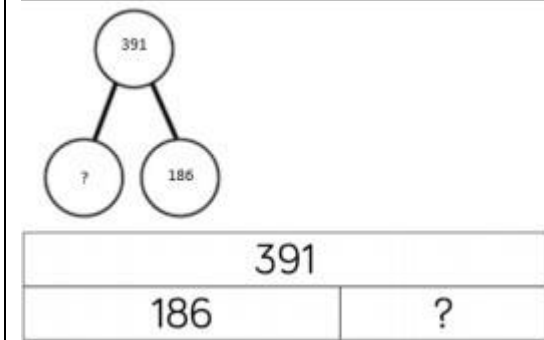
$$48 - 27 = \text{(No regrouping)}$$

At Broughton we use lots of different techniques on a number line including: Counting in 1s, Subtracting the 10s then the ones, targeting 10. Subtracting 10 then taking some away if the number is near 10 etc.

			<p>b) Column method:</p> $48 - 7 =$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>4</td><td>8</td></tr> <tr><td>-</td><td></td><td>7</td></tr> <tr><td></td><td>4</td><td>1</td></tr> </table>		4	8	-		7		4	1					
	4	8															
-		7															
	4	1															
6	<p>Column method (using base 10 and having to exchange):</p> <p style="text-align: center;">45-26</p> <ol style="list-style-type: none"> 1) Start by partitioning 45 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens. 	<p>Represent the base 10 pictorially, remembering to show the exchange.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>10s</td><td>1s</td></tr> <tr><td>1+10</td><td>5</td></tr> </table> <p>We would also show this on a number line.</p>	10s	1s	1+10	5	<p>Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td></td><td>4</td><td>11</td></tr> <tr><td>-</td><td>2</td><td>6</td></tr> <tr><td></td><td>1</td><td>5</td></tr> </table>		4	11	-	2	6		1	5	<p>This is a key concept that can take some time to embed so we practise it in a variety of ways and keep revisiting it. (Number lines, part whole models, bar models etc)</p>
10s	1s																
1+10	5																
	4	11															
-	2	6															
	1	5															
7	<p>Column method using place value counters.</p> <p>234 - 88</p> 	<p>Represent the place value counters pictorially; remembering to show what has been exchanged.</p> 	<p>Formal column method. Children must understand what has happened when they have crossed out digits.</p> $\begin{array}{r} 234 \\ - 88 \\ \hline 146 \end{array}$	<p>After this point the children would try with larger numbers and decimals.</p>													

As teachers we understand that there is a range of different ways to ask a question to make sure that children understand the key concepts fully. Below is a table of different ways we might ask a subtraction question.

Conceptual Understanding: Different ways to ask children to solve 391-186



Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\square = 391 - 186$$

391

-186

What is 186 less than 391?

Missing digit calculations

$$\begin{array}{r} 39\square \\ - \square\square 6 \\ \hline \square 0 5 \end{array}$$

Addition and Subtraction: Helpful hints and tips

Throughout this progression of skills children will experience solving word problems, using it in different contexts (such as measure and money), real life problems and multi-step problems.

Mental Maths that children need to be able to recall in addition and subtraction to support the methods above:

KS1:

Know the pairs of numbers for all numbers up to 10 and use these to recall number facts for numbers up to 20 □

- Begin to use number bonds to solve addition problems (quickly adding the units) e.g. $44 + 27$ ($4+6 = 10$, $10+1 = 11$, $60+11 = 71$) □ To be able to partition numbers in a variety of ways (strong understanding of number and place value. Know, recognise and use doubles and halves to 10 at least.

□

- Be able to recognise the relationship between addition and subtraction e.g. $26 - 24$ (counting up would be a more efficient strategy)

KS2:

- Know the number bonds to 100 and apply this to larger numbers
- Count on and back in whole numbers, fractions and decimals (hundreds, tens, tenths etc. – ensure they understand what happens when you cross a tens number)
- To understand which calculations they are able to do mentally and which they will need a formal written method for.
- ‘Compensation’: When adding 8 or 9, add 10 and take away 1 or 2. (near multiples of 10)
- Know, recognise and use doubles and halves (whole numbers and decimals)
- Be able to recognise the relationship between addition and subtraction e.g. $7012 - 6991$ (counting up would be a more efficient strategy)

Multiplication and Division Progression of Skills

Key language: Odd, even, count in twos, threes, fives, count in tens (forwards from/backwards from) How many times? Lots of, groups of, once, twice, three times, five times, Multiple of, times, multiply, multiply by Repeated addition, Array, row, column, double, halve, share, share equally, group in pairs, threes, etc. , equal groups of divide, divided by, left, left over, **Product**, **Multiples of four, eight, fifty and one hundred**, **Scale up**, **Multiplication facts (up to 12x12)**, **Division facts**, **Inverse**, **Derive**, **Factor pairs**, **Composite numbers**, **prime number**, **prime factors**, **square number**, **cubed number**, **Formal written method**, **Order of operations**, **Common factors**, **common multiples**

Year 1/2: black words

Year 3: red words

Year 4: blue words

Year 5: green words

Year 6: Purple words **Expectations**

(National Curriculum Statements):

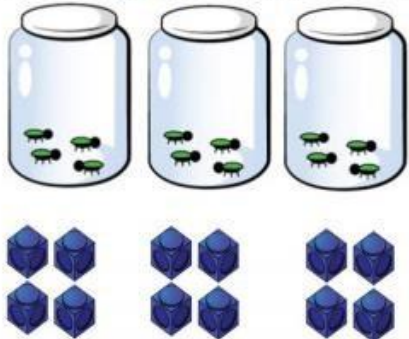
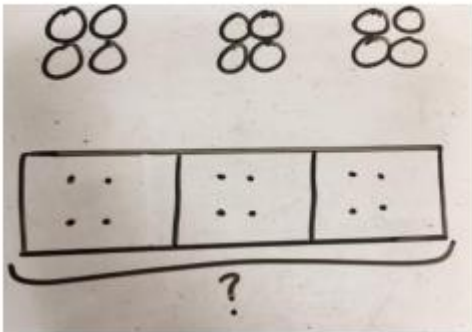
Year R: -Solve problems, including doubling, halving and sharing.		
Year 1	Year 2	Year 3
<ul style="list-style-type: none"> -solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. - count in multiples of twos, fives and tens 	<ul style="list-style-type: none"> - Count in steps of 2, 3, and 5 from 0, and in 10s from any number, forward and backward - Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers - Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs -Show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot -Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts. 	<ul style="list-style-type: none"> -Count from 0 in multiples of 4, 8, 50 and 100 - Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables -Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods -Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.
Year 4	Year 5	Year 6
<ul style="list-style-type: none"> -Count in multiples of 6, 7, 9, 25 and 1 000 - Recall multiplication and division facts for multiplication tables up to 12×12 -Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers - Recognise and use factor pairs and commutativity in mental calculations -Multiply two-digit and three-digit numbers by a onedigit number using formal written layout -Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects. 	<ul style="list-style-type: none"> - <i>Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000</i> -Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. -Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers -Establish whether a number up to 100 is prime and recall prime numbers up to 19 -Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers -Multiply and divide numbers mentally drawing upon known facts -Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context 	<ul style="list-style-type: none"> -Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication -Divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context -Use <i>written division methods in cases where the answer has up to two decimal places</i> -Identify common factors, common multiples and prime numbers -Use <i>common factors to simplify fractions; use common multiples to express fractions in the same denomination</i>

-Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
 -Recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)
 -Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes -Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
 -Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.



- Calculate, estimate and compare volume of cubes and cuboids
 using standard units, including centimetre cubed (cm³) and ³₃ cubic metres (m³), and extending to other units such as mm³ and km³
 - Use their knowledge of the order of operations to carry out calculations involving the four operations
 - Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

If children are identified as struggling to learn multiplication and division facts we put in interventions where appropriate.

Progression of skills in Multiplication

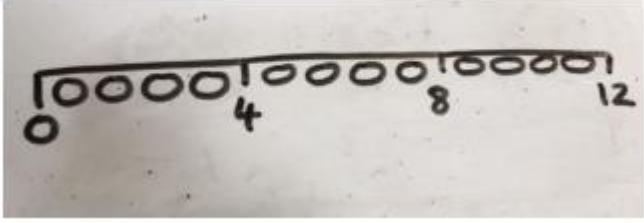
Step	Concrete	Pictorial	Abstract	
1	<p>Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group.</p> 	<p>Children to represent the practical resources in a picture and use a bar model.</p> 	<p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>	<p>We make sure children are confident in counting in these patterns at this point.</p>

2 **Number lines to show repeated groups-**
 3×4

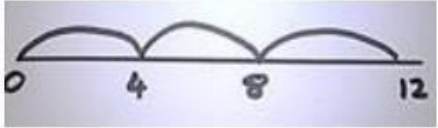
Cuisenaire rods can be used too.

Represent this pictorially alongside a number line e.g.:




Abstract number line showing three jumps of four.

$3 \times 4 = 12$




The children would also continue to use lots of resources to show this.

3 **Use arrays to illustrate commutativity** counters and other objects can also be used.
 $2 \times 5 = 5 \times 2$

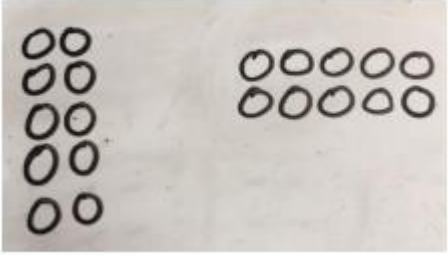


2 lots of 5



5 lots of 2

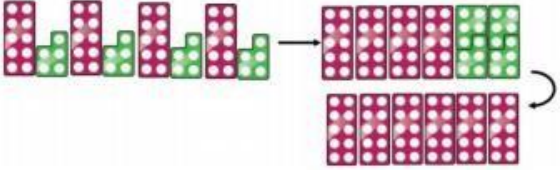
Children to represent the arrays pictorially.



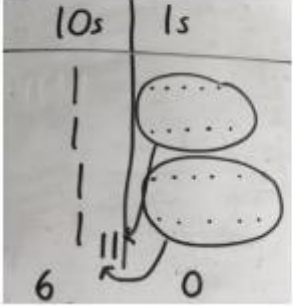
Children to be able to use an array to write a range of calculations e.g.

$10 = 2 \times 5$
 $5 \times 2 = 10$
 $2 + 2 + 2 + 2 + 2 = 10$
 $10 = 5 + 5$

4 **Partition to multiply** using Numicon, base 10 or Cuisenaire rods.
 4×15

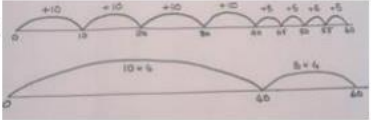


Children to represent the concrete manipulatives pictorially.



Children to be encouraged to show the steps they have taken:

a) **Number line:**



b) **Grid Method:**

$4 \times 15 =$

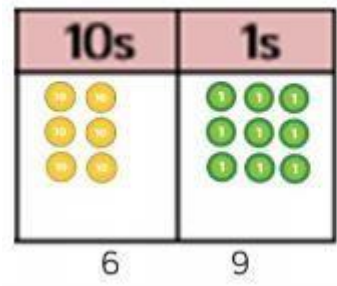
	10	5	
4	40	20	= 60

$14 \times 15 =$

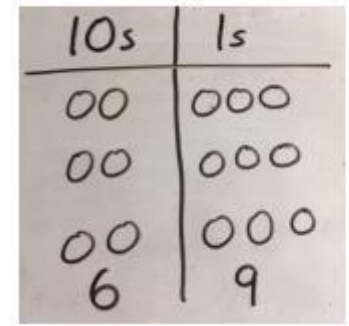
	10	5	
10	100	50	= 150
4	40	20	= 60
			210

At this point, bar models and pv counters are used.

5 **Formal column method** with place value counters (base 10 can also be used.) 3×23



Children to represent the counters pictorially.

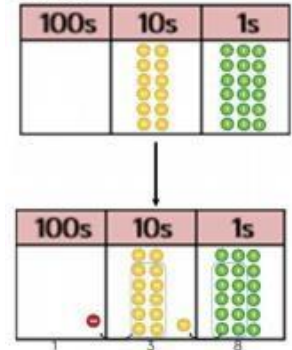


Children to record what it is they are doing to show understanding.

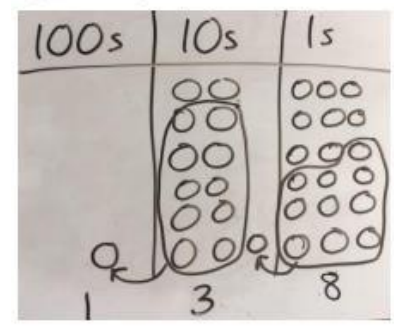
3×23 $3 \times 20 = 60$
 $20 \quad 3$ $3 \times 3 = 9$
 $60 + 9 = 69$

$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

6 **Formal column method** with place value counters. 6×23



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

$6 \times 23 =$

$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ 11 \end{array}$$

After this step we would expect the children to progress onto larger numbers, decimals etc.

7 When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc., they should be confident with the abstract:


To get 744 children have solved 6×124 .
 To get 2480 they have solved 20×124 .

$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ 11 \end{array}$$

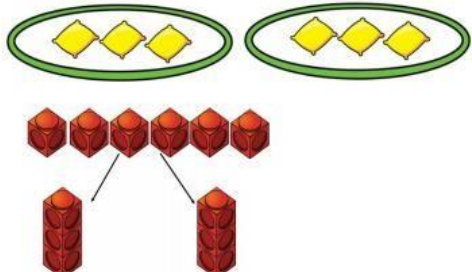
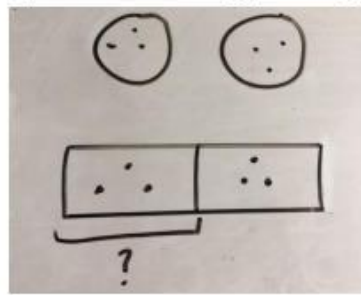
Answer: 3224

As teachers we understand that there is a range of different ways to ask a question to make sure that children understand the key concepts fully. Below is a table of different ways we might ask Multiplication question.

Conceptual Understanding: Different ways to ask children to solve 6x23

	<p>Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?</p> <p>With the counters, prove that $6 \times 23 = 138$</p>	<p>Find the product of 6 and 23</p> <p>$6 \times 23 =$</p> <p>$\square = 6 \times 23$</p> $\begin{array}{r} 6 \quad 23 \\ \times 23 \quad \times 6 \\ \hline \end{array}$	<p>What is the calculation? What is the product?</p> <table border="1" data-bbox="1612 263 2116 502"> <thead> <tr> <th>100s</th> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td></td> <td>6 columns of 2 yellow counters each</td> <td>6 columns of 3 green counters each</td> </tr> </tbody> </table>	100s	10s	1s		6 columns of 2 yellow counters each	6 columns of 3 green counters each
100s	10s	1s							
	6 columns of 2 yellow counters each	6 columns of 3 green counters each							

Progression of skills in Division

Step	Concrete	Pictorial	Abstract			
1	<p>Sharing using a range of objects. $6 \div 2$</p> 	<p>Represent the sharing pictorially.</p> 	<p>$6 \div 2 = 3$</p> <table border="1" data-bbox="1467 901 1926 981"> <tr> <td>3</td> <td>3</td> </tr> </table> <p>Children should also be encouraged to use their 2 times tables facts.</p>	3	3	<p>At this point we do a lot of different practical examples- using sweets, cakes, toys etc.</p>
3	3					

2 **Repeated subtraction** using Cuisenaire rods above a ruler.
 $6 \div 2$

3 groups of 2

Children to represent repeated subtraction pictorially.

Abstract number line to represent the equal groups that have been subtracted.

3 **2d + 1d with remainders** using lollipop sticks. Cuisenaire rods, above a ruler can also be used.
 $13 \div 4$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.

There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

There are 3 whole squares, with 1 left over.

$13 \div 4 = 3$ remainder 1

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

4 **Sharing using place value counters.**
 $42 \div 3 = 14$

Children to represent the place value counters pictorially.

$48 \div 4 =$

4 tens $\div 4 = 1$ ten 8 ones $\div 4 = 2$ ones
 $10 + 2 = 12$

Children to be able to make sense of the place value counters and write calculations to show the process.

$42 \div 3$
 $42 = 30 + 12$
 $30 \div 3 = 10$
 $12 \div 3 = 4$
 $10 + 4 = 14$

Chunking Method:

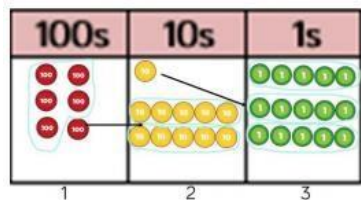
$42 \div 3 =$

At this point we introduce chunking on a number line.

5

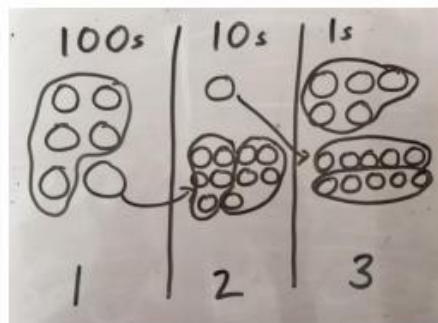
Short division using place value counters to group.

$615 \div 5$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

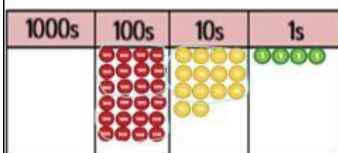
Represent the place value counters pictorially.



Children to the calculation using the short division scaffold.

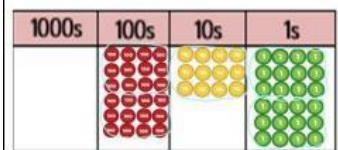
$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

6



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

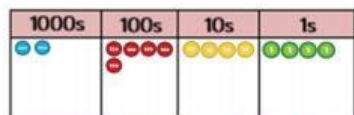


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

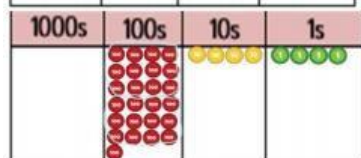
$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Long division using place value counters

$2544 \div 12$



We can't group 2 thousands into groups of 12 so will exchange them.



We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

$$\begin{array}{r} 0 \\ 12 \overline{) 2544} \end{array}$$

Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I

grouped and the 2 is how many tens I have left.

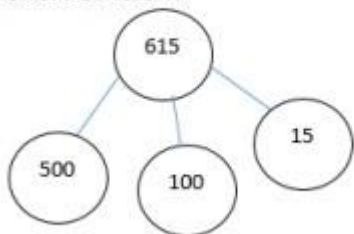
$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

As teachers we understand that there is a range of different ways to ask a question to make sure that children understand the key concepts fully. Below is a table of different ways we might ask Division question.

Conceptual Understanding: Different ways to ask children to solve $615 \div 5 =$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

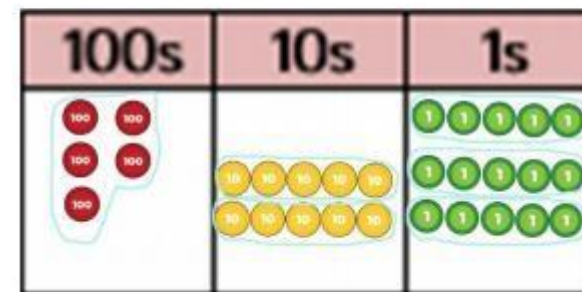
615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?
What is the answer?



Multiplication and Division: Helpful hints and tips

Throughout this progression of skills children will experience solving word problems, using it in different contexts (such as measure and money), real life problems and multistep problems.

Mental Maths that children need to be able to recall in multiplication and division to support the methods above:

KS1:

- To count in steps of 2, 3, 5 and 10
 - To rapidly recall number facts for the 2,3,5 and 10 times tables
 - To use doubles and near doubles / halves and near halves
 - show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- KS2:
- To count in steps of 4, 6, 7, 8, 9, 11 and 12
 - To rapidly recall number facts for all the times tables.
 - To use doubles and near doubles / halves and near halves.